

Natural Neutrino Dark Energy

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ABSTRACT: A new class of neutrino dark energy models is presented. The new models are characterized by the lack of exotic particles or couplings that violate the standard model symmetry. It is shown that these models lead to several concrete predictions for the dark energy equation of state, as well as possible effects on the cosmic structure formation. These predictions, can be verified (or disproved) with future experiments. At this point, the strongest constraints on these models are obtained from big bang nucleosynthesis, and lead to new bounds on the mass of the lightest neutrino.

KEYWORDS: [Neutrino](#), [Dark Energy](#), [BBN](#).

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1. Introduction

It has been over a decade since observations of supernovae revealed that the expansion of the universe seems to be accelerating [1]. Today in addition to supernova observations, the acceleration of the universe, is supported by data from WMAP [2], SDSS [3] and more [4]. The source of this acceleration, the so called "dark energy" (DE), remains a mystery, since it cannot be any of the well known cosmological components. The only previously known energy source that can play the role of DE is the vacuum (or zero-point) energy. The magnitude of the zero-point energy density depends on the QFT cutoff. However, any natural cutoff, generates an enormous vacuum energy, up to 120 orders of magnitude greater than the value needed to account for the observed acceleration of the universe. Thus, in addition to the strange nature of DE, one must account for its "tiny" value ($\rho_{\text{DE}} \sim (10^{-3} \text{eV})^4$), the same value responsible for the "cosmic coincidence" - the fact that today, DE is of the same order of magnitude as the cold dark matter density, despite the fact that the ratio between them changes as the cosmic scale factor cubed.

Countless theories have been presented to explain the source of cosmic acceleration and its value. Most of them include either modified gravity [5, 6] or exotic sources of energy [7, 8], deviating far from the standard model (SM). Since strong deviations from known physics are usually accompanied by numerous degrees of freedom, testing for these theories is very difficult, in spite of the advancing observational constraints on DE [2, 3, 4, 9]. In addition, most theories seem to be unable to naturally generate the correct value for DE, usually leaving it as another degree of freedom, resulting in the need for fine-tuning. Finding ourselves in such a vast landscape of alternative theories, emphasizes the need for a theory with minimal deviations from known physics and preferably no fine tuning.

neutrino masses are the closest known fundamental energy scale to the scale of DE [10, 11]. This implies that neutrinos might be connected to DE, and that a theory with an underlying link between them would have less additional degrees of freedom.

This idea is not new, and various methods to link neutrino physics to DE, have been investigated [12, 13, 14, 15, 16, 17]. One of the leading models for neutrino DE is mass varying neutrinos (MAVANS) [18, 19]. This class of models includes coupling between neutrinos and a scalar field (the Acceleron). The DE "tracks" the matter density to a critical point, when the temperature of the cosmic background and the mass of the neutrino are comparable, and then becomes constant and generates the acceleration we witness today. All the models, mentioned above, have specific issues, but one issue is common to all of them: It is unnatural for the neutrino to be the only source of DE. For example, in the case of MAVANS, there is no reason for the Acceleron to couple only to the neutrino. Being a neutral scalar particle, coupling to a neutrino must imply coupling to the electron (and by the same token, the muon and the tau). A natural model of neutrino DE cannot include such an exotic coupling that distinguishes the neutrino from the other leptons, thus, breaking the SM symmetry.

2. Constructing the Model

In order to construct a natural neutrino DE (NNDE) model, it is essential to find a quality that separates the neutrino from the other leptons. The obvious would be the lack of electric charge. Models, in which the DE component is coupled to Majoranna mass terms, naturally account for the neutrino being the only source of DE. However, since Majoranna masses are typically very high, we lose the correlation between the mass and the DE scale and thus, the entire motivation behind neutrino DE. Therefore, it is necessary to find a different quality that would distinguish the neutrino.

The three neutrino species, appear to be the lightest fermions. Particularly, the lightest neutrino is the only known massive particle that *may* still be relativistic in the cosmic background. The heavier neutrinos are not relativistic (based on neutrino oscillations [10]), but they are much closer than any other known fermion. Thus, we can base a NNDE model on the idea that relativistic particles contribute to DE, while non-relativistic ones do not. We construct a model, such that the energy density is

$$\rho_{\text{DE}}(a) = \sum_{i=\text{particles}} U(\xi_i(a), \chi_i(a), \dots) . \quad (2.1)$$

Here $\xi_i(a), \chi_i(a), \dots$ are scalar functions of certain parameters of the cosmic ensemble (referring to the particle species i). $\xi_i(a)$ and $\chi_i(a)$ are almost constant in the ultra-relativistic regime and diverge as the kinetic energy of the particles drops to zero (alternatively, as the cosmic scale factor $a \rightarrow \infty$). The function U must decay to zero as $\xi(a), \chi(a), \dots$ diverge, so there is no contribution to DE from non-relativistic particles. Also, to correlate DE to particle masses, $U \sim m^4$ in the ultra-relativistic regime, where m is the particle mass. In eq. (2.1), it was not specified, whether we sum over all elementary particles, fermions or only leptons. This is to be determined in a specific model and is irrelevant at this point. Eq. (2.1) is not restricted to a specific model and guaranties the following:

- No exotic couplings that violate particle physics symmetries are necessary, since DE couples evenly to every particle.
- At any given time, DE is dominated by the heaviest relativistic particles. Heavier, non relativistic particles, have no contribution. Therefore,
 - Today, DE is dominated by the (lightest?) neutrino.
 - In the past, heavier particles contributed as well and the DE density was greater. This, will be referred to as primordial DE.

The form of eq. (2.1) is therefore sufficient to explain why the neutrino is the only one to contribute to DE today.

To proceed, it is necessary to find all the possible parameters $\xi(a), \chi(a), \dots$ that satisfy the above mentioned conditions. The mean energy E must be a factor in all the parameters, since it defines the relativistic and non-relativistic regimes. E scales with expansion oppositely to what we have defined; it is constant in the non relativistic regime and diverges in the ultra-relativistic one. Therefore the energy must be suppressed by a quantity that constantly grows with expansion. Thus, two simple parameters come to mind, $\xi = El$ and $\chi = E/T$. Where l is the mean distance between the particles and T is the temperature of the cosmic background (T is not necessarily the CMB temperature. For example, for neutrinos $T = T_{\text{CMB}} (4/11)^{1/3}$)

3. NNDE Phenomenology

At this point, we can find an expression for the DE equation of state (EOS),

$$1 + w_{\text{DE}} = -\frac{1}{3} \frac{a}{\rho_{\text{DE}}} \frac{\partial \rho_{\text{DE}}}{\partial a} . \quad (3.1)$$

Assuming, only a single neutrino species gives a significant contribution, substituting eq. (2.1) into eq. (3.1), we have

$$1 + w_{\text{DE}} = -\frac{1}{3} \frac{a}{U} \left(\frac{\partial U}{\partial \xi} \frac{\partial \xi}{\partial a} + \frac{\partial U}{\partial \chi} \frac{\partial \chi}{\partial a} \right) . \quad (3.2)$$

For cosmic neutrinos [11, 17], we can write

$$E = \frac{\int \frac{4\pi p^2 dp}{(2\pi)^3} \frac{\sqrt{m^2 + p^2}}{1 + \exp(p/T)}}{\int \frac{4\pi p^2 dp}{(2\pi)^3} \frac{1}{1 + \exp(p/T)}} , \quad (3.3)$$

$$l = \frac{1}{\left(2 \int \frac{4\pi p^2 dp}{(2\pi)^3} \frac{1}{1 + \exp(p/T)} \right)^{1/3}} . \quad (3.4)$$

Model	$w_a(w_0)$
NNDE	$w_a = -2(1 + w_0)$
basic MaVaNs	$w_a = -3(1 + w_0)^2$
Vacuum fluctuations	$w_a = -3(1 + w_0)^2$
Chaplygin gas	$w_a = 3(1 + w_0) (1 - (1 + w_0)^2)$
DGP	$w_a = 3(1 + w_0) \left(1 - 2(1 + w_0) \left(1 + \sqrt{-w_0(1 + w_0)}\right)\right)$
Quintessence Thawing	$-3(1 + w_0) \leq w_a \leq -(1 + w_0)$
Quintessence Freezing	$-0.2w_0(1 + w_0) \lesssim w_a \leq -3w_0(1 + w_0)$

Table 1: The linear evolution approximation of the DE EOS, for different models. From top to bottom: NNDE, a simple MaVaN model [18], Vacuum energy model [15], Chaplygin gas [8], DGP [5], quintessence thawing and quintessence freezing [7].

Expanding the square root in eq. (3.3) via $\sqrt{m^2 + p^2} \simeq p + m^2/2p$ and substituting it along with eq. (3.4) into eq. (3.2), we have

$$w_{\text{DE}} \simeq -1 - \left(\beta_\xi \frac{\partial \ln U}{\partial \xi} + \beta_\chi \frac{\partial \ln U}{\partial \chi} \right) \left(\frac{m}{T} \right)^2, \quad (3.5)$$

where,

$$\beta_\xi = \frac{1}{27} \left(\frac{\pi^8}{12\zeta(3)^4} \right)^{1/3} \simeq 0.268 \quad \text{and} \quad \beta_\chi = \frac{\pi^2}{54\zeta(3)} \simeq 0.152$$

are numerical coefficients. U and its derivatives are functions of ξ and χ , both of which are constant to first order, in the ultra-relativistic regime. Therefore, in the ultra-relativistic regime $T \sim a^{-1}$ is the fastest varying quantity in eq. (3.5), it follows that to first order, $1 + w_{\text{DE}} \sim a^2$. Expanding eq. (3.5) via powers of $1 - a$ we have,

$$w_{\text{DE}} = w_0 + w_a(1 - a) + w_2(1 - a)^2, \quad (3.6)$$

with

$$w_a = -2w_2 = -2(1 + w_0), \quad (3.7)$$

and

$$w_0 = -1 - \left(\beta_\xi \frac{\partial \ln U}{\partial \xi} + \beta_\chi \frac{\partial \ln U}{\partial \chi} \right) \left(\frac{m}{T_0} \right)^2. \quad (3.8)$$

Where T_0 is the temperature at the present time. Eq. (3.6) is valid even if w_{DE} deviates strongly from -1, as long as the ratio m/T is small, and thus the particles are relativistic.

Verifying the relation $w_a(w_0)$ is a good, parameter independent, method to test a variety of DE models. Table 1 shows the linear evolution approximation of the DE EOS, for different models. Some of these models are plotted in figure 1. One can clearly see the distinctive behavior of the different models.

As shown, NNDE corresponds to the thawing range of parameters for $w > -1$. A way to distinguish the two may be to analyze observational data, using eq. (3.6) rather than its linear approximation.

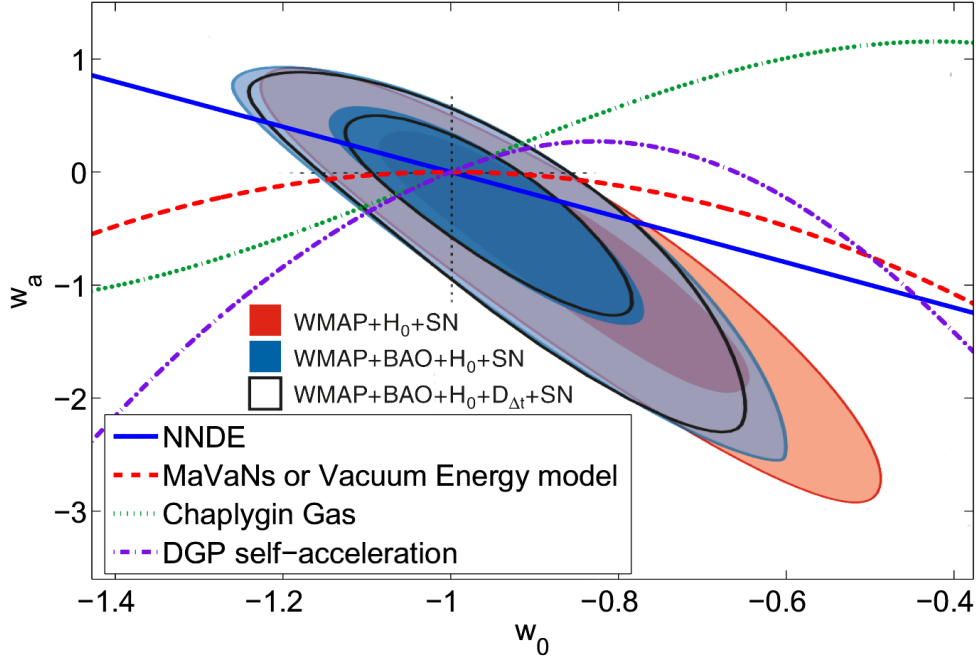


Figure 1: The linear evolution approximation of the DE EOS, for different models. The plotted contours signify the observational constraints, taken from [2]. Within the observational constraints, the different models are clearly distinguishable, aside from the DGP and Chaplygin gas curves.

Another important prediction of NNDE models lies within its initial formulation. There is no natural way for neutrino to induce DE, without heavier leptons inducing it as well. We have eliminated the contribution of heavier leptons by constructing a model where only relativistic particles have significant contributions. However, this means that in the early universe, heavier leptons did contribute (primordial DE). The further we go back in time, the more contributions we have. In order for us to discuss the cosmic history in the presence of NNDE, we must choose a specific model, that is to determine a specific function $U(\xi, \chi)$. As a toy model, let us use

$$U = Cm^4 \exp(-\xi), \quad (3.9)$$

where, C is a constant. I emphasize that the toy model is used merely for exact calculations and all the conclusions below are model independent.

Since, unlike neutrinos, many fermions stay coupled to the cosmic plasma in the era where $T \sim m$ (which is the era where the DE of the specific particle dominates the expansion), the exact contribution of those fermions is slightly different. Figures 2 and 3 show $\exp(-\xi)$ and $\exp(-\chi)$ as a function of m/T for coupled and decoupled fermions. It is visible that the general behavior of the model for coupled fermions, remains the same.

Figure 4 shows the expansion history, in terms of the Hubble plot, for Λ CDM vs NNDE. On the Hubble plot, the mass hierarchy of the different leptons is clearly visible.

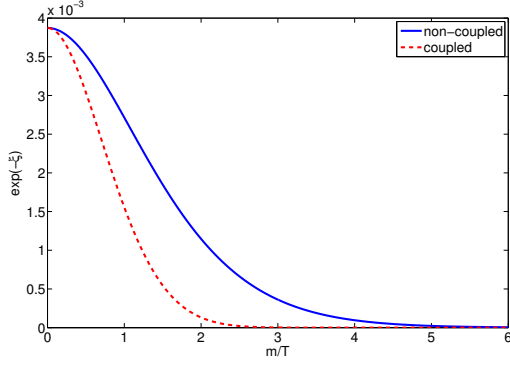


Figure 2: $\exp(-\xi)$ vs m/T for couples and decoupled fermions.

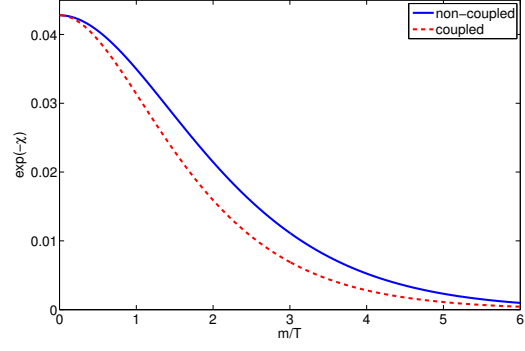


Figure 3: $\exp(-\chi)$ vs m/T for couples and decoupled fermions.

The "staircase" form of the Hubble plot is in correlation with the masses of the different leptons¹. The first DE "step" (the one closest to the present epoch) is due to the μ neutrino, the next are due to the τ neutrino, then the electron and so on.

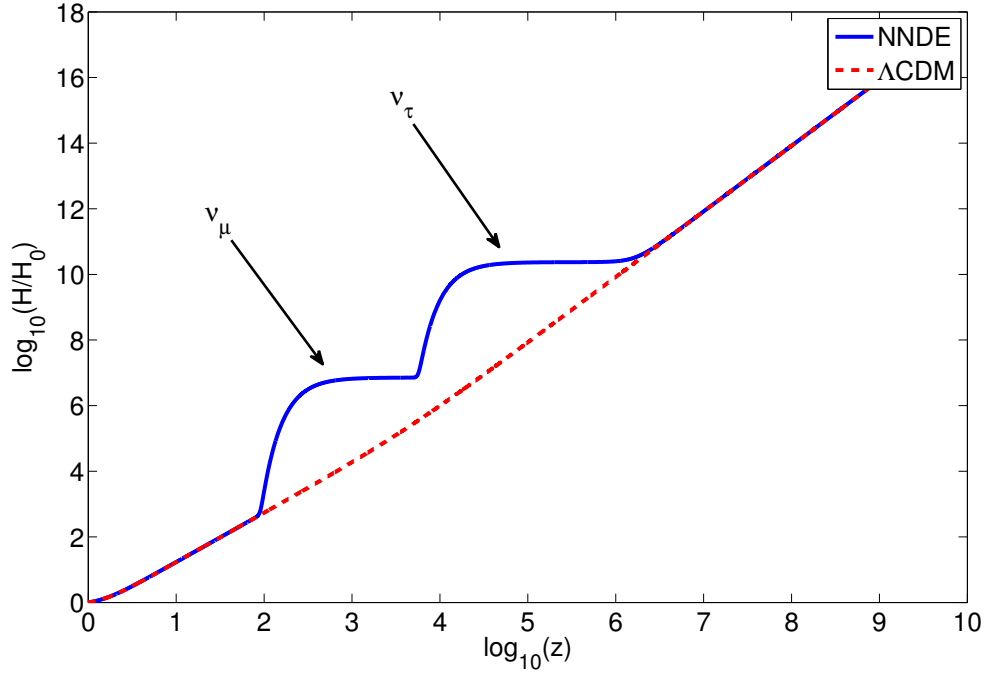


Figure 4: The Hubble plot for NNDE and Λ CDM models.

¹The neutrino masses are unknown so this is a schematic plot.

4. Constraining NNDE Using Big Bang Nucleosynthesis

4.1 NNDE Constraints

Other than using the relation dictated by Eq. (3.7), It is difficult (although not impossible) to constrain the NNDE model via usual astrophysical observations, including CMB. The reason is, that NNDE phenomenology does not differ much from other DE models, including the cosmological constant. The major difference lies in the primordial DE. However, since we do not know the neutrino masses, it is difficult to state, at which phase in the 'recent' expansion of the universe, do primordial DE effects show themselves.

In fact, a well posed constraint on the NNDE comes from big bang nucleosynthesis (BBN). Since the BBN freeze-out occurs at a temperature of about $\sim 1MeV$ [20], it should be strongly affected by the electron-induced DE. Unlike the neutrino masses, we cannot maneuver our way around this issue. Both the BBN ratios [21] and electron mass are measured with good precision and a significant variation of either is out of the question.

There are two ways to ensure that the presence of the electron would not have significant effects on the BBN, without abandoning the principles of NNDE:

- The first way is to construct a model, where the contribution of any particle to DE decays before it becomes non relativistic (To be referred as type I NNDE). This would assure that by the time of the BBN freeze-out, the electron-induced DE is negligible.
- Finally, it is possible to construct a model such that the contribution of any particle to DE, is suppressed by a factor of several orders of magnitude, so it is still proportional to m^4 , but lower (To be referred as type II NNDE). This can render the electron-induced DE low enough so it would not significantly affect the BBN.

To proceed, we modify the toy model from Eq. (3.9) to

$$U = Cm^4 \exp(-g\xi). \quad (4.1)$$

Here, g is added as a parameter which tunes the energy scale, beyond which a particle no longer contributes to DE.

4.2 Type I NNDE

In this model, to assure that the electron-induced DE does not affect the BBN, we require g to be large enough, so that $\exp(-gEl)$ quickly decays for temperatures $T < 3MeV$. Using Eq. (3.5) obtained for the EOS, the above requirement can be written as

$$g \gg \frac{1}{\beta_\xi} \left(\frac{3MeV}{m_e} \right)^2 \sim 100. \quad (4.2)$$

At the same time, we require $1 + w_0 \ll 1$. Thus, the neutrino mass must be small enough, so that

$$m_\nu \ll \frac{T_\nu}{\sqrt{g\beta_\xi}} \simeq 3 \cdot 10^{-5} eV, \quad (4.3)$$

where $T_\nu = T_{\text{CMB}}(4/11)^{1/3}$. Figure 5, presents the constraints on the neutrino mass and the parameter g , obtained from BBN freeze-out (neutron to proton ratio) and the DE EOS. The BBN freeze-out ratio, was calculated numerically using a generalization of the model presented in [20],

$$X_n = \int_0^\infty \frac{dy}{2y^2 (1 + \cosh(1/y))} \times \exp \left(-\lambda \int_0^y dy' H^{-1} y'^2 (y' + 0.25)^2 (1 + \exp(-1/y')) \right). \quad (4.4)$$

Here, H is the Hubble constant, $\lambda \simeq 3.26 \text{sec}^{-1}$ and $y = T/Q$, where, $Q \simeq 1.29 \text{MeV}$ is the difference between neutron and proton mass. This model gives a good approximation of the freeze-out ratio, but more importantly, of its sensitivity to the cosmic expansion. To cancel out the overall error of this model, only the difference $X_n(\text{NNDE}) - X_n(\Lambda\text{CDM})$ is used.

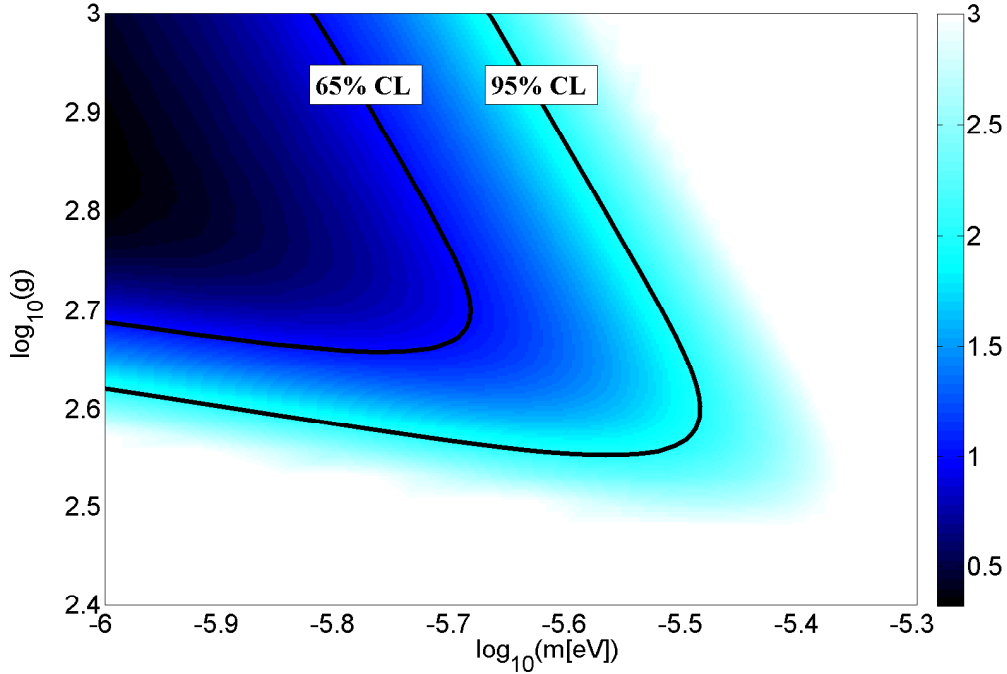


Figure 5: type I NNDE constraint on g and the neutrino mass from the DE EOS and BBN measurements. The color-bar signifies the sigma confidence level.

The 95% confidence level constraints of this model², are therefore (see Figure 5),

$$\begin{aligned} m_\nu &\lesssim 3 \cdot 10^{-6} \text{eV}, \\ g &\gtrsim 350. \end{aligned} \quad (4.5)$$

²The constraint on g is irrelevant, since it is a model related parameter. Using a different model, would have resulted in different constraints on its parameters, however, *it would not have a significant effect on the neutrino mass constraint.*

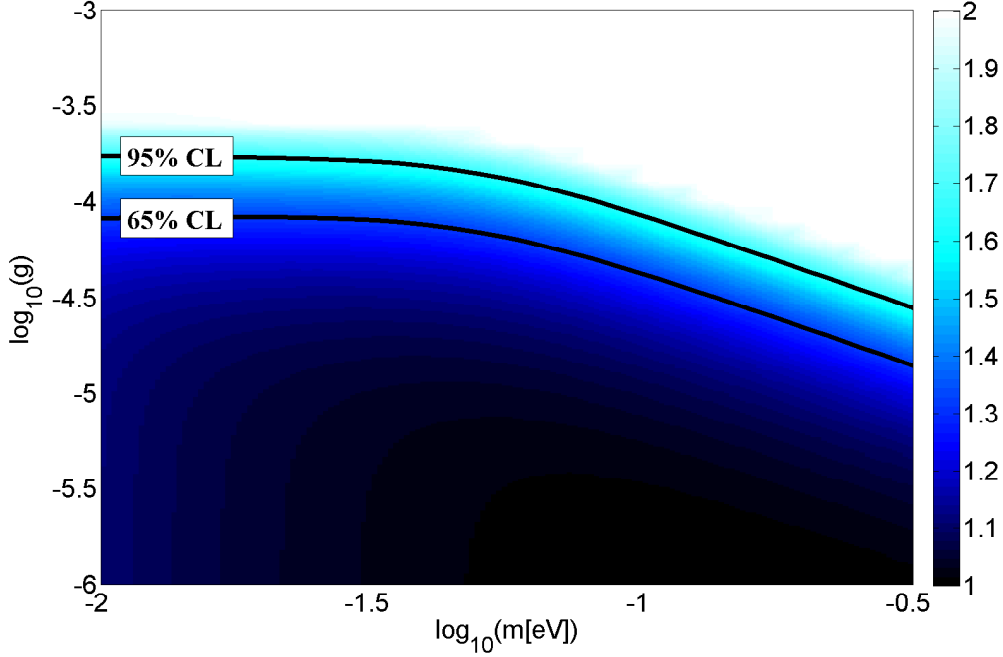


Figure 6: type II NNDE constraint on g and the lightest neutrino mass from the DE EOS and BBN measurements. The color-bar signifies the sigma confidence level. In this case, there is no constraint on the mass of the lightest neutrino.

This is of the same order of magnitude as the value obtained in [17]. It is worth noting that according to these constraints on the lightest neutrino mass and using the best estimate we have for the neutrino square mass difference (obtained from neutrino oscillations [10]), the muon neutrino mass is $m_{\nu\mu} \simeq 8.7 \cdot 10^{-3} eV$. Therefore, in accordance with the previously noted fact, that a particle does not contribute to DE, once the universe cools beyond a certain temperature $T \propto m$, and knowing that the lightest neutrino dominates comoving expansion at red-shifts $z \sim 1$ the muon-neutrino-induced DE, dominated the expansion of the universe, up to red-shifts of

$$z_{\nu\mu} \gtrsim \frac{m_{\nu\mu}}{m_{\nu e}} \simeq 3000 \sim z_{\text{eq}}. \quad (4.6)$$

Here z_{eq} is the red-shift of the radiation-matter equality. This is a remarkable result, because it tells us that the primordial DE would not have significantly affected structure growth, and yet leaves room for subtle effects, that will perhaps be measured in the future.

4.3 Type II NNDE

In this case, we require the electron-induced DE to be subdominant (compared to the radiation energy density) all the way up to the end of the freeze-out era $T^* \sim 200 keV$, so

$$Cm_e^4 \ll \kappa a_{\text{rad}} T^{*4}, \quad (4.7)$$

with a_{rad} being the radiation density constant and $\kappa \simeq 7.25$ is the number of radiative degrees of freedom, at the corresponding temperature. Since, today we have $Cm_\nu^4 = \rho_{\text{DE}}$, eq. 4.7 implies, for the neutrino mass,

$$m_\nu > \frac{m_e}{T^*} \left(\frac{\rho_{\text{DE}}}{\kappa a_{\text{rad}}} \right)^{1/4} \simeq 4 \cdot 10^{-3} eV. \quad (4.8)$$

For this mass range, we can no longer assume only the lightest neutrino contributes. Therefore, for this model, the calculations include all three neutrino generations. The relations between the different neutrino masses, were taken to be the nominal values from [10],

$$\begin{aligned} \Delta m_{\mu e}^2 &= 7.6 \cdot 10^{-5} eV^2, \\ \Delta m_{\tau \mu}^2 &= 2.4 \cdot 10^{-3} eV^2. \end{aligned} \quad (4.9)$$

Figure 6, presents the constraints of this model on the lightest neutrino mass and the parameter g . There are no constraints on the neutrino mass for this model. However, we see that the maximal allowed value of g is inversely proportional to $m_{\nu\tau}$. In this case, all three neutrino species, are involved in generating today's DE. Therefore, there are no available particles to induce primordial DE in proximity to the radiation-matter equality, and thus, no observable effects on the cosmic structure formation.

5. Summary and Discussion

A natural model for neutrino dark energy (NNDE), free of exotic particles or couplings, was presented. Basing the theory only on naturalness and the assumption that today's DE is driven by the neutrino, several unique predictions were derived. A specific relation $w_a(w_0)$ governing the linear evolution approximation of the EOS, follows from NNDE. Verifying the above relation, can strongly support this model. Although, quintessence thawing models can produce similar $w_a(w_0)$ dependance, distinguishing between the two may be possible via a more advanced analysis of observations, using second order expansion of the EOS.

NNDE also requires the existence of primordial DE, driven by heavier particles, dominating the expansion of the early universe. To prevent the electron-induced primordial DE from affecting the BBN, two methods were proposed. The first, entitled type I NNDE, is to assure that the electron-induced DE decays prior to the BBN freeze-out phase. The second method, entitled type II NNDE, is to assure that the magnitude of the electron-induced DE, is small enough to have little effect of the BBN. Using the above methods, constraints on NNDE are placed.

Type I NNDE, provides new constraints on the mass of the lightest neutrino, requiring $m_\nu < 3 \cdot 10^{-6} eV$. In addition, type I NNDE, predicts relic primordial DE, induced by the muon neutrino, in proximity to the radiation-matter equality phase. This would mildly affect structure formation and the CMB. Observing these effects could prove as the main finger-print of the NNDE.

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